KISIELEWICZ MODEL FOR EQUATION 1516

JOSE BROX

$$x = y^2(x(xy)) \qquad \text{Equation 1516}$$

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1. Model rules

(a) $$x \cdot x = 2^x$$

(b) $$x \cdot 2^x = 3^x 5^x$$

(c) $$x \cdot (2^x 3^y) = 3^x 5^y$$

(d) $$2^{2^{2^x} 3^y} \cdot 2^x = 2^{3^y 5^x}$$

(e) $$2^{2^{2^x} 3^y} \cdot (2^{2^{3^y 5^x} 3^{2^x}}) = 2^{3^y 5^x}$$

(f) $$2^{3^x 5^y} \cdot (2^{2^y} 3^x) = 2^y$$

(g) $$2^{2^x} \cdot (3^x 5^x) = x$$

(h) $$2^x \cdot (3^{2^x} 5^x) = 3^{2^x} 5^x$$

(i) $$2^y \cdot (3^x 5^y) = x \text{ (except if } y = z, x = 2^z)$$

(j) $$2^{2^y 3^y} \cdot (3^x 5^y) = x$$

(k) $$x \cdot (3^x 5^y) = 3^x 5^y \text{ (except if } x = 2^y \text{ or } x = 2^{2^y 3^y})$$

(l) $$2^{2^x 3^y} \cdot (3^x 5^y) = x$$

(m) $$2^{3^{3^x 5^y} 5^y} \cdot x = 2^y$$

(n) $$x \cdot y = 2^x 3^y \text{ otherwise.}$$

Ambiguities: We analyze only those cases which are not completely obvious.

- (a),(**??**), (**??**) and (b) are disjoint, since $x, 2^x, 2^{2^x}, 2^{2^{2^x} 3^x}$ are pairwise distinct.
- (c) and (f) are disjoint, since (putting $x = 2^\alpha$ in (c)) $\alpha \neq 3^y 5^\alpha$.

- (g) is disjoint with (h),(i),(j),(k),(l) since $2^{2x} \neq x \neq 2^x \neq 2^x 3^x$.
- (h) is disjoint with (j),(l) since $x \neq 2^x$.
- (i) is disjoint with (j),(l) since $2^y 3^y \neq y \neq 2^x 3^y$.
- (j) and (l) are disjoint except if $x = y$, in which case they both resolve to the same element x.
- If (k) may apply, then (h), (i) and (j) may apply too.
- (k) and (l) are disjoint, since $x \neq 2^{2^x 3^y}$.
- Even if in (m) we have a free x as right factor, it does not have any ambiguities with the previous rules in which the right factor is of the form 2^α, $2^\alpha 3^\beta$ or $3^\alpha 5^\beta$. E.g., consider a possible ambiguity with (f), in which $x = 2^{2^\alpha} 3^\beta$. This then would imply that $\beta = 3^{2^{2^{2^\alpha 3^\beta}} 5^\alpha}$, which is impossible.

2. Validity of the model

(1) In the generic case:

a) **First level:**

$$y^2 = 2^y, \quad xy = 2^x 3^y.$$

- (a) applies to y^2.
- No rule other than (n) applies to xy.

b) **Second level:**

$$x \cdot (2^x 3^y) = 3^x 5^y.$$

- (c) applies.

c) **Third level:**

$$2^y \cdot (3^x 5^y) = x.$$

- (i) applies.

(2) If $x = y$:

a) **First level:**

$$x^2 = 2^x.$$

- (a) applies to x^2 twice.

b) **Second level:**

$$x \cdot 2^x = 3^x 5^x.$$

- (b) applies.

c) **Third level:**

$$2^x \cdot (3^x 5^x) = x.$$

- (i) applies.

(3) If $y = 2^x$:

a) **First level:**

$$y^2 = 2^y, \quad xy = x \cdot 2^x = 3^x 5^x.$$

- (a) applies to y^2, (b) applies to xy.

b) **Second level:**

$$x \cdot 3^x 5^x = 3^x 5^x.$$

- (h),(i) do not apply since $x \neq 2^x$.
- (j),(l) do not apply since $x \neq 2^{2^x 3^x}$.
- (k) applies.

c) **Third level:**

$$2^y \cdot (3^x 5^x) = 2^{2^x} \cdot (3^x 5^x) = x.$$

- (i) applies.

(4) If $y = 2^x 3^z$:

a) **First level:**

$$y^2 = 2^y, \quad xy = x \cdot 2^x 3^z = 3^x 5^z.$$

- (a) applies to y^2, (c) applies to xy.

b) **Second level:**

$$x \cdot 3^x 5^z.$$

- (l) does not apply since $x \neq 2^{2^x 3^y}$.
- If $x = 2^z$ then (h) applies:

$$2^z \cdot 3^{2^z} 5^z = 3^{2^z} 5^z = 3^x 5^z.$$

- If $x \neq 2^z$ then (k) applies:

$$x \cdot 3^x 5^z = 3^x 5^z.$$

c) **Thirld level:** (l) applies:

$$2^y \cdot 3^x 5^z = 2^{2^x 3^z} \cdot 3^x 5^z = x.$$

(5) If $x = 2^{3^u 5^v}$, $y = 2^{2^v} 3^u$:

a) **First level:**

$$y^2 = 2^y, \quad xy = 2^{3^u 5^v} \cdot 2^{2^v} 3^u = 2^v.$$

- (a) applies to y^2, (f) applies to xy.

b) **Second level:**

$$2^{3^u 5^v} \cdot 2^v.$$

- If $u = 3^{2^v} 5^v$ then (m) applies:

$$x \cdot 2^v = 2^{3^{3^{2^v} 5^v} 5^v} \cdot 2^v = 2^v.$$

- If $u \neq 3^{2^v} 5^v$ then (n) applies:

$$2^{3^u 5^v} \cdot 2^v = 2^{2^{3^u 5^v}} 3^{2^v}.$$

c) **Third level:**

- If $u = 3^{2^v} 5^v$ then (d) applies:

$$2^y \cdot 2^v = 2^{2^{2^v} 3^u} \cdot 2^v = 2^{3^u 5^v} = x.$$

- If $u \neq 3^{2^v} 5^v$ then (e) applies:

$$2^y \cdot 2^{2^{3^u 5^v}} 3^{2^v} = 2^{2^{2^v} 3^u} \cdot 2^{2^{3^u 5^v}} 3^{2^v} = 2^{3^u 5^v} = x.$$

Email address: `josebrox@uva.es`