Operad laws in a virtual equipment

Functors

$A \longrightarrow B$

Profunctors

$A \nrightarrow B$

2-Cell

$$\begin{array}{ccc} A & \nrightarrow & B \\ \downarrow & \Downarrow & \downarrow \\ C & \nrightarrow & D \end{array}$$

Unit

$$\begin{array}{ccc} & C & \\ & \Downarrow & \\ A & \nrightarrow & B \end{array}$$

Binatural transformation

$$\begin{array}{ccccc} A & \nrightarrow & B & \nrightarrow & C \\ \downarrow & & \Downarrow & & \downarrow \\ D & & \longrightarrow & & E \end{array}$$

there could have n inputs!

Let (T, η, μ) be a monad (in a suitable sense)

An operad on a profunctor $TO \overset{\Lambda}{\nrightarrow} O$ (source: TO; target: O)

consists of a unit and a binatural transformation

$$\begin{array}{ccc} O & & \\ {\scriptstyle \eta_O}\swarrow & \Downarrow id & \\ TO & \overset{\Lambda}{\nrightarrow} & O \end{array}$$

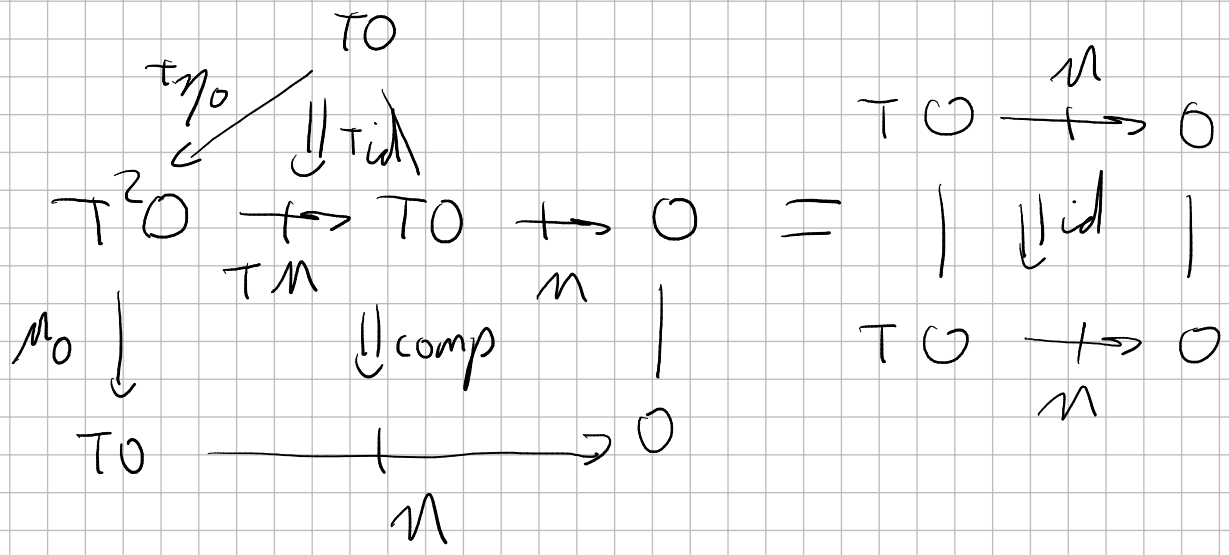
$$\begin{array}{ccccc} T^2O & \nrightarrow & TO & \nrightarrow & O \\ {\scriptstyle \Lambda}\downarrow & & \Downarrow comp & & \downarrow \\ TO & & \longrightarrow & & O \end{array}$$

s.t.

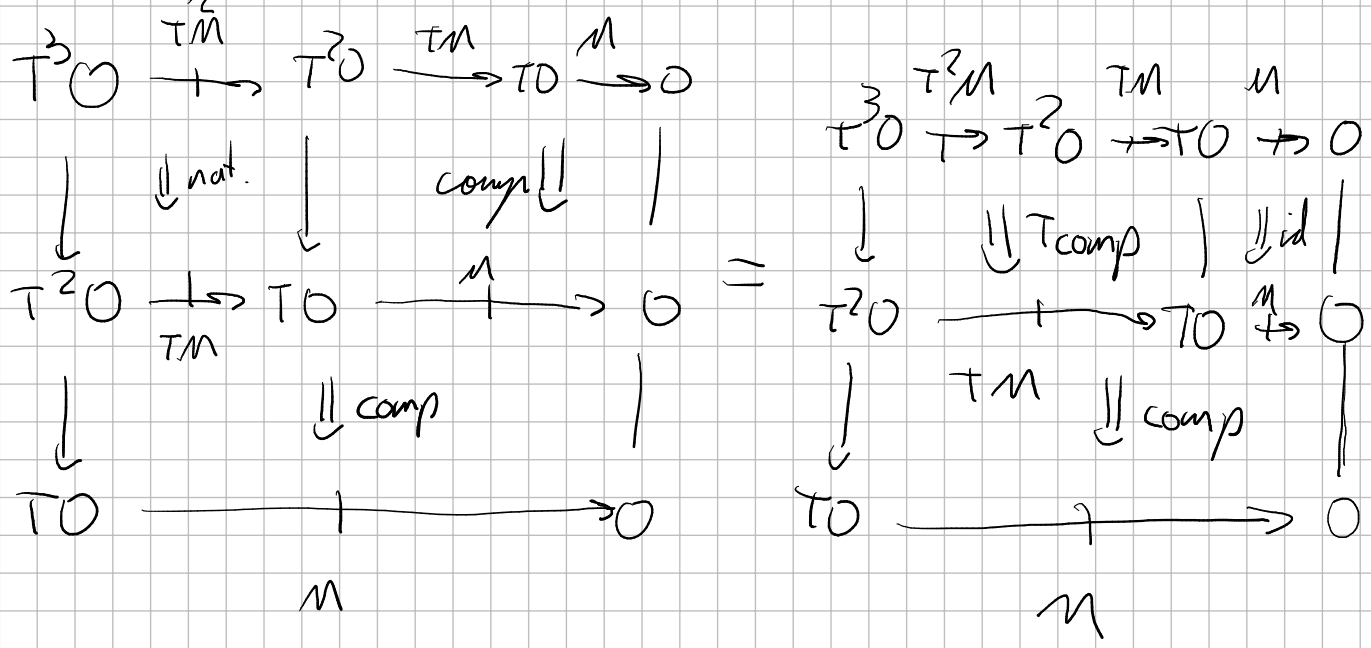
$$\begin{array}{ccccc} & & TO & \overset{\Lambda}{\nrightarrow} & O \\ & {\scriptstyle \eta_{TO}}\swarrow & \Downarrow naturality & & \\ T^2O & & \nrightarrow & & TO \\ {\scriptstyle T\Lambda}\downarrow & & & & \downarrow {\scriptstyle \Lambda} \\ TO & & \nrightarrow & & O \end{array} \quad = \quad \begin{array}{ccc} TO & \overset{\Lambda}{\nrightarrow} & O \\ \downarrow & & \downarrow \\ TO & \overset{\Lambda}{\nrightarrow} & O \end{array}$$

$$\begin{array}{ccccc} & & O & & \\ & {\scriptstyle \eta_O}\swarrow & \Downarrow id & \searrow^{\eta_O} & \\ TO & & \nrightarrow & & TO \\ {\scriptstyle \Lambda}\downarrow & & \Downarrow comp & & \downarrow {\scriptstyle \Lambda} \\ TO & & \longrightarrow & & O \end{array} \quad = \quad \begin{array}{ccc} TO & \overset{\Lambda}{\nrightarrow} & O \\ \downarrow & & \downarrow \\ TO & \overset{\Lambda}{\nrightarrow} & O \end{array}$$

2)


TO
Tη_O
‖ Tid
T²O
TO
O
Tμ
μ
μ_O
‖ comp
TO
O
μ
=
TO
μ
O
‖ id
TO
O
μ

3)


T³O
T²μ
T²O
Tμ
TO
μ
O
‖ nat.
‖ comp
T²O
Tμ
TO
μ
O
‖ comp
TO
O
μ
=
T³O
T²μ
Tμ
μ
T²O
TO
O
‖ Tcomp
‖ id
T²O
TO
O
Tμ
‖ comp
TO
O
μ