Let U be the real-valued function $f(x)$, where x a vector, $x = \langle x_1, x_2 \rangle$. Thus, f is a function of two variables. We can write the total derivative of f which is:

$$df = \left(\frac{\partial f}{\partial x_1}\right)_{x_2} dx_1 + \left(\frac{\partial f}{\partial x_2}\right)_{x_1} dx_2.$$

Suppose that the first derivative of f is known for each variable, such that $\left(\frac{\partial f}{\partial x_1}\right)_{x_2} = A(x)$ and $\left(\frac{\partial f}{\partial x_2}\right)_{x_1} = B(x)$, where A and B are real valued functions. From the symmetry of the second derivative, we know that:

$$\frac{\partial}{\partial x_2}\left(\left(\frac{\partial f}{\partial x_1}\right)_{x_2}\right)_{x_1} = \frac{\partial}{\partial x_1}\left(\left(\frac{\partial f}{\partial x_2}\right)_{x_1}\right)_{x_2}$$

Using this identity along with the A and B, we can then write that:

$$\frac{\partial}{\partial x_2}(A)_{x_1} = \frac{\partial}{\partial x_1}(B)_{x_2}$$

Which can be written as

$$\left(\frac{\partial A}{\partial x_2}\right)_{x_1} = \left(\frac{\partial B}{\partial x_1}\right)_{x_2}$$