6.36. Definition. A point M is **maximal** in a subset S of a poset P if M is a member of S and no member of S is bigger,

$$M \in S \,\&\, (\forall x \in S)[M \leq x \Longrightarrow M = x].$$

A point m is **minimal** in S if it is a member of S and no member of S is smaller,

$$m \in S \,\&\, (\forall x \in S)[x \leq m \Longrightarrow x = m].$$

x6.6. Find in the poset of Figure **6.2** a subset S which has a maximal element but no maximum and another subset S' which has a minimal element but no minimum.

***x6.7.** Every finite, non-empty subset of an arbitrary poset P has at least one maximal and one minimal member.

***x6.8.** A finite poset P is inductive if and only if it has a least element.

An important notion in computer science is that of a *stream*, for example the stream of bytes in a file transmitted over the telephone lines to my home computer from the University of Athens CYBER. A stream is basically a sequence, but it may be *infinite*, in the idealized case; *terminated*, if after some stage an end-of-file signal comes and my machine knows that the transmission is done; or *stalled*, if after some stage the bytes stop coming, without warning, perhaps because the CYBER died or the telephone connection was interrupted.[16]

6.37. Definition. For each set A, we fix some $t \notin A$ (for example, the object $\boldsymbol{r}(A)$ of (3-4)) and we define the **streams from** A by:

$$\text{Streams}(A) =_{\text{df}} \{\sigma : \mathbb{N} \rightharpoonup A \cup \{t\} \mid (\forall i < j)[\sigma(j)\downarrow \Longrightarrow [\sigma(i)\downarrow \,\&\, \sigma(i) \neq t]]\}.$$

We call a stream σ terminated or **convergent** if for some n, $\sigma(n) = t$, in which case, by the definition Domain$(\sigma) = [0, n+1)$; **infinite** if Domain$(\sigma) = \mathbb{N}$; and **stalled** if Domain(σ) is a finite, initial segment of $\mathbb{N}$ but σ does not take on the *terminating value* t. The infinite and stalled streams together are called **divergent**.

x6.9. For each set A, the set of streams Streams(A) is an inductive poset under the natural, partial ordering $\sqsubseteq$, where, as for strings,

$$\sigma \sqsubseteq \tau \iff_{\text{df}} \sigma \subseteq \tau. \tag{6-22}$$

What are its maximal elements?

x6.10. The **concatenation** $\sigma \star \tau$ of two streams is defined so that if σ is divergent, then $\sigma \star \tau = \sigma$ and if σ is convergent with domain $[0, n+1)$, then

$$i < n \Longrightarrow (\sigma \star \tau)(i) = \sigma(i), \quad (\sigma \star \tau)(n+i) = \tau(i).$$

[16]Well, the CYBER has died completely since the first edition of these Notes, but the newer machines and more robust telephone lines still quit unexpectedly on some occasions!