This is a translation into linear maps of Knuth's "computational" reformulation [1] of Huang's proof of the Sensitivity Conjecture [2].

Let $(V_n)_{n\geq 0}$ be the sequence of $\mathbb{R}$-vector spaces defined inductively by

$$V_0 = \mathbb{R}, \quad V_{n+1} = V_n \oplus V_n.$$

Each space V_n has an obvious basis $(e_p)_{p\in Q^n}$ indexed on $Q^n = \texttt{bool}^n$. Let $(\varepsilon_p : V_n \to \mathbb{R})_{p\in Q^n}$ be the dual linear functionals.

Let $f_n : V_n \to V_n$ be the linear map defined inductively by

$$f_0 = 0, \quad f_{n+1}(v, w) = (f_n v + w, v - f_n w).$$

By induction $f_n^2 v = nv$. Moreover $|\varepsilon_q(f_n e_p)|$ is 1 if p and q are adjacent vertices of the hypercube graph Q^n (which we denote by $p \sim q$) and 0 otherwise.

Now let $g_n : V_{n-1} \to V_n$ be defined by

$$g_n v = (f_{n-1} v + \sqrt{n} v, v).$$

Then g_n is injective, so its image is a subspace of V_n of dimension $\dim V_n = 2^{n-1}$. Furthermore for any $w = g_n v$ in this image we have

$$\begin{aligned} f_n w &= f_n(f_{n-1} v + \sqrt{n} v, v) \\ &= (f_{n-1} f_{n-1} v + \sqrt{n} f_{n-1} v + v, f_{n-1} v + \sqrt{n} v - f_{n-1} v) \\ &= (\sqrt{n} f_{n-1} v + nv, \sqrt{n} v) \\ &= \sqrt{n} w. \end{aligned}$$

Let H be a subset of Q^n of cardinality at least $2^{n-1} + 1$. The restriction of the basis of V_n to H spans a subspace W of V_n of dimension at least $2^{n-1} + 1$. Since the image of g_n has dimension 2^{n-1} and $2^{n-1} + 1 + 2^{n-1} > 2^n$, we can find some nonzero vector y belonging to both W and the image of g_n, so that $f_n y = \sqrt{n} y$.

Now write $y = \sum_{p\in H} y_p e_p$ and let $q \in H$ be an index with $|y_q|$ maximal. Then

$$\begin{aligned} |\sqrt{n} y_q| &= |\varepsilon_q(\sqrt{n} y)| \\ &= \big|\varepsilon_q\big(f_n\big(\sum_{p\in H} y_p e_p\big)\big)\big| \\ &\leq \sum_{p\in H} |\varepsilon_q(f_n e_p)||y_p| \\ &= \sum_{p\in H, p\sim q} |y_p| \\ &\leq \sum_{p\in H, p\sim q} |y_q| \\ &= |\{\, p \in H \mid p \sim q \,\}||y_q| \end{aligned}$$

and therefore q has at least $\sqrt{n}$ neighbors in H.

[1]: `https://www.cs.stanford.edu/~knuth/papers/huang.pdf`
[2]: `http://www.mathcs.emory.edu/~hhuan30/papers/sensitivity_1.pdf`