

Final goal:

$$\begin{aligned}
 q &= \frac{a * P}{(1 - b * P)(1 - b * P + C * P)} \\
 a \& b \& C \& P \in \mathbb{R} \\
 P &\geq 0 \\
 \text{Axiom 1 : } A &= \sum_{i=0}^{+\infty} \theta_i \\
 A \& \theta \in \mathbb{R} \\
 i &\in \mathbb{N} \\
 \theta &\geq 0 \\
 \& \theta &\leq 1 \\
 \text{Axiom 2 : } V_{ads} &= V_0 * \sum_{i=0}^{+\infty} i * \theta_i \\
 V_{ads} \& V_0 \& \theta \in \mathbb{R} \\
 i &\in \mathbb{N} \\
 V_0 &> 0 \\
 \text{Axiom 3 : } q &= \frac{V_{ads}}{A} \tag{1} \\
 q &\in \mathbb{R} \\
 \text{Axiom 4 : } r_{ci-1} &= k_i * P * \theta_{i-1} \\
 k_i &\in \mathbb{R} \\
 k_i &\geq 0 \\
 \text{Axiom 5 : } r_{ei} &= k_{-i} * \theta_i \\
 k_{-i} &\in \mathbb{R} \\
 k_{-i} &\leq 0 \\
 \text{Axiom 6 : } r_{ei} &= r_{Ci-1} \\
 \text{Axiom 7 : } k_{-2} &= \dots = k_{-i} = k_{des} \\
 k_{des} &\geq 0 \\
 \text{Axiom 8 : } k_2 &= \dots = k_i = k_{ads} \\
 k_{des} &\geq 0 \\
 \text{Axiom 9 : } \theta_i &= C * x^i * \theta_0 \\
 \theta_0 \& x &\in \mathbb{R} \\
 \theta_0 &\geq 0 \\
 \& \theta_0 &\leq 1
 \end{aligned}$$

Proving :

$$\text{applying Axiom 3 : } q = \frac{V_{ads}}{A}$$

$$\text{Substituting Axiom 1 \& 2 at Axiom 3 : } q = \frac{V_0 * \sum_{i=0}^{+\infty} i * \theta_i}{\sum_{i=0}^{+\infty} \theta_i}$$

Dividing both side of equation by  $V_0$

and Changing starting point of summation from zero to one :

$$\frac{q}{V_0} = \frac{0 * \theta_0 + \sum_{i=1}^{+\infty} i * \theta_i}{\theta_0 + \sum_{i=1}^{+\infty} \theta_i}$$

Substituting  $\theta_i$  :

$$\frac{q}{V_0} = \frac{0 * \theta_0 + \sum_{i=1}^{+\infty} i * C * x^i * \theta_0}{\theta_0 + \sum_{i=1}^{+\infty} C * x^i * \theta_0}$$

Taking out constant values like  $C$  &  $\theta_0$  from summation :

$$\frac{q}{V_0} = \frac{C * \theta_0 * \sum_{i=1}^{+\infty} i * x^i}{\theta_0 * (1 + C * \sum_{i=1}^{+\infty} x^i)}$$

cancelling  $\theta_0$

$$\frac{q}{V_0} = \frac{C * \sum_{i=1}^{+\infty} i * x^i}{(1 + C * \sum_{i=1}^{+\infty} x^i)}$$

$$\text{using this assumption : } \sum_{i=1}^{+\infty} x_i = \frac{x}{(1-x)}$$

$$\sum_{i=1}^{+\infty} ix_i = \frac{x}{(1-x)^2}$$

Substituting them at the above equation :

$$\frac{q}{V_0} = \frac{\frac{C*x}{(1-x)^2}}{1 + \frac{C*x}{(1-x)}}$$

$$\frac{q}{V_0} = \frac{\frac{C*x}{(1-x)^2}}{\frac{1-x+C*x}{1-x}}$$

Cancelling  $(1-x)$  :

$$\frac{q}{V_0} = \frac{C * x}{(1-x)(1-x+C*x)}$$

$$x = \frac{k_{ads}}{k_{des}} * P$$

$$y = \frac{k_1}{k_{-1}} * P$$

$$C = \frac{y}{x}$$

(2)

Substituting  $C$  &  $x$  :

$$C = \frac{y}{x} = \frac{\frac{k_1}{k_{-1}}}{\frac{k_{ads}}{k_{des}}}$$

$$\frac{q}{V_0} = \frac{\frac{k_1}{k_{-1}} * P}{(1 - \frac{k_{ads}}{k_{des}} * P)(1 - \frac{k_{ads}}{k_{des}} * P + \frac{k_1}{k_{-1}} * P)}$$

$$q = \frac{\frac{k_1}{k_{-1}} * V_0 * P}{(1 - \frac{k_{ads}}{k_{des}} * P)(1 - \frac{k_{ads}}{k_{des}} * P + \frac{k_1}{k_{-1}} * P)}$$

Assigning the following to constants  $a$  &  $b$

$$a = \frac{k_1}{k_{-1}} * V_0$$

$$b = \frac{k_{ads}}{k_{des}}$$

$$q = \frac{a * P}{(1 - b * P)(1 - b * P + C * P)} \quad (3)$$

Type of Variables:

$A$  &  $V_{ads}$  &  $q$  &  $\theta$  &  $\theta_0$  &  $V_0$  &  $P$  &  $k_1$  &  $k_{-1}$  &  $k_{ads}$  &  $k_{des}$  &  $k_2$  &  $k_{-2}$  are real numbers  
 $i$  : natural number

Constraints:

$\theta \geq 0$  &  $\theta \leq 1$  &  $k_{ads} > 0$  &  $k_{des} > 0$  &  $k_1 > 0$  &  $k_{-1} > 0$  &  $k_2 > 0$  &  $k_{-2} > 0$  &  $V_0 > 0$  &  $P \geq 0$