Self-Learned Formula Synthesis in Set Theory

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This work has been supported by the European Research Council (ERC) grant AI4REASON no. 649043 under the EU-H2020 programme.

September 14, 2020

Motivation

Understanding formulas is important for theorem proving.

How do we figure out what a formula means?

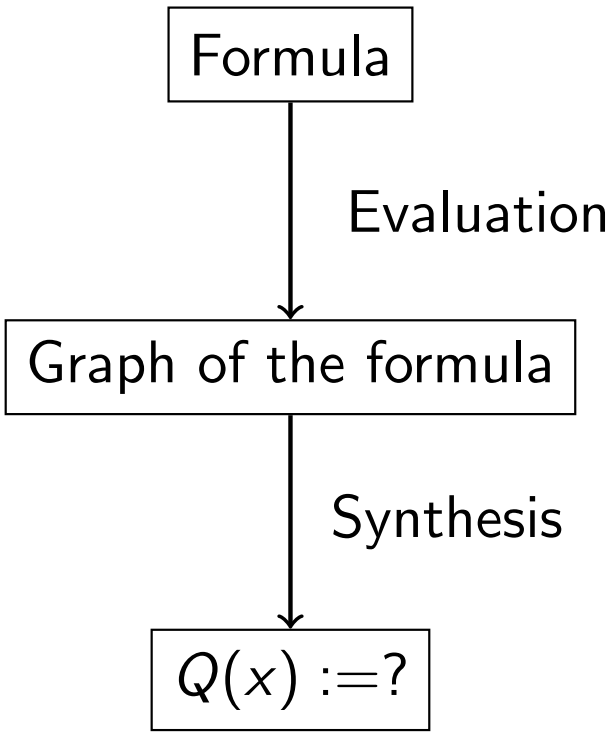
Example formula

What is the meaning of this set-theoretical formula?

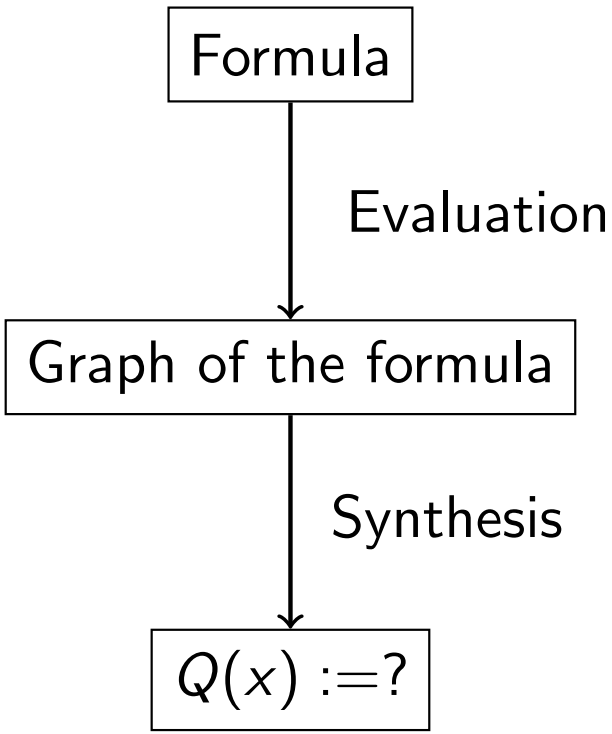
$$\exists y \in x.\ x \not\subseteq \wp(y)$$

There exists a set y member of x, such that
x is not a subset of the power set of y.

Outline



Outline



$$P(x) := \exists y \in x.\ x \not\subseteq \wp(y)$$

$$P(a_0), P(a_1), \ldots, P(a_{63})$$

Vocabulary

Terms s, t :

$$x, y, z, \ldots \qquad \varnothing \qquad \wp(t) \qquad \{t\} \qquad s \cup t$$

Atomic formulas φ, ψ :

$$s \in t \qquad s \notin t \qquad s \subseteq t \qquad s \not\subseteq t \qquad s = t \qquad s \neq t$$

Composite formulas φ, ψ :

$$\varphi \Rightarrow \psi \qquad \varphi \wedge \psi$$

$$\forall x \in s.\ \varphi \quad \forall x \subseteq s.\ \varphi \qquad \exists x \in s.\ \varphi \quad \exists x \subseteq s.\ \varphi$$

Example formula: understanding a special case

$$P(x) := \exists y \in x.\ x \not\subseteq \wp(y)$$

$$P(\varnothing) := \exists y \in \varnothing.\ \varnothing \not\subseteq \wp(y)$$

$P(\varnothing)$ is false.

What is a finite set?

Ground terms a,b:

$$\varnothing \qquad \wp(a) \qquad \{a\} \qquad a \cup b$$

The set $\{\varnothing, \{\varnothing\}\}$ can be constructed as $\{\varnothing\} \cup \{\{\varnothing\}\}$

Enumerating finite sets ($f : \mathbb{N} \to$ finite sets)

$$f(10) \to f(0101) \to \{f(1), f(3)\} \to \{f(1), f(11)\} \to$$

$$\{\{f(0)\}, \{f(0), f(1)\}\} \to \{\varnothing, \{\varnothing, \{\varnothing\}\}\}$$

Idea: position of the 1s in the inverted binary encoding.

Example formula: truth values on initial sets

$P(x) := \exists y \in x.\ x \not\subseteq \wp(y)$

$P(0)$ *is false*.
$P(1)$ *is false*.
$P(01)$ *is false*.
$P(11)$ *is true*.
$P(001)$ *is false*.
$P(101)$ *is true*.
$P(011)$ *is true*.
$P(111)$ *is true*.
$P(0001)$ *is false*.
$\ldots P(63)$

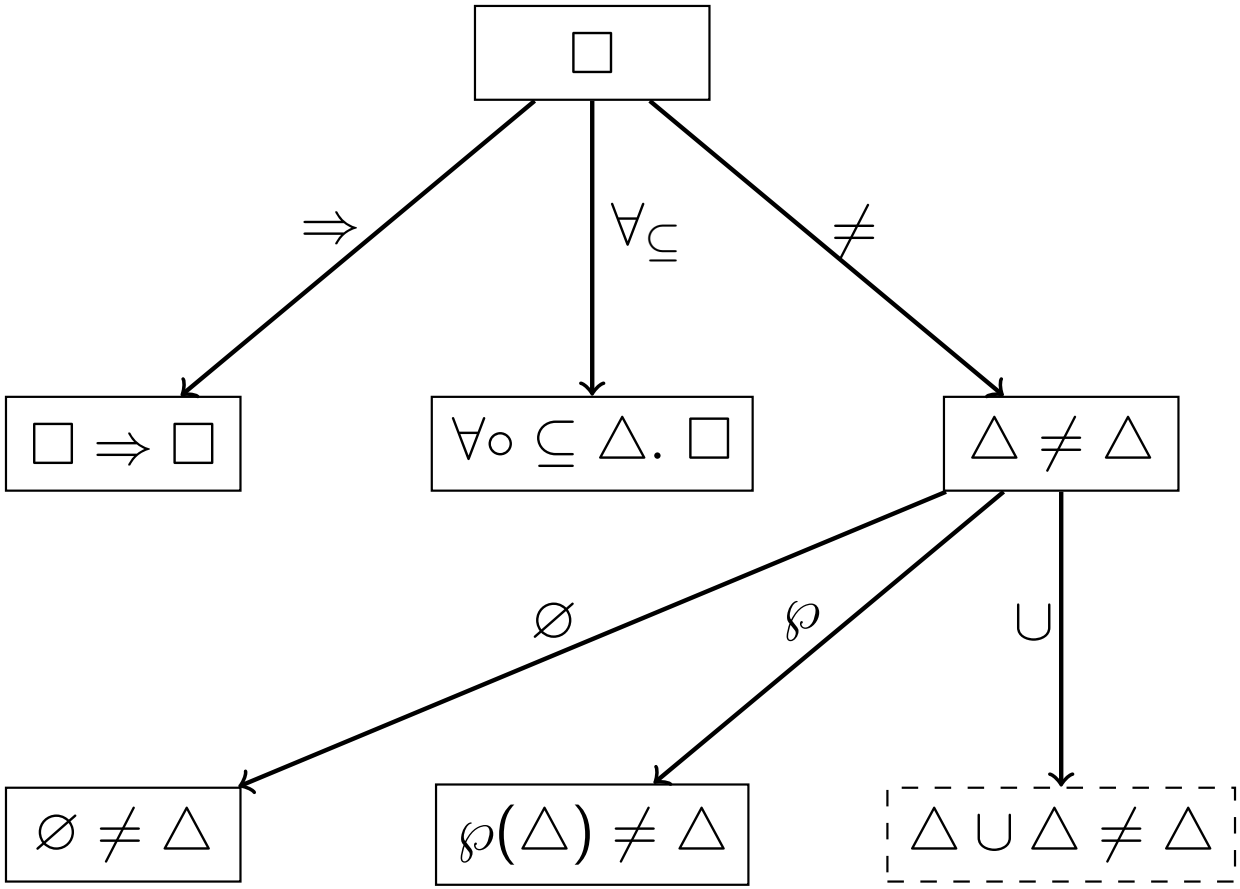
Graph of P : $FFFTFTTTF\ldots$

Synthesis problem

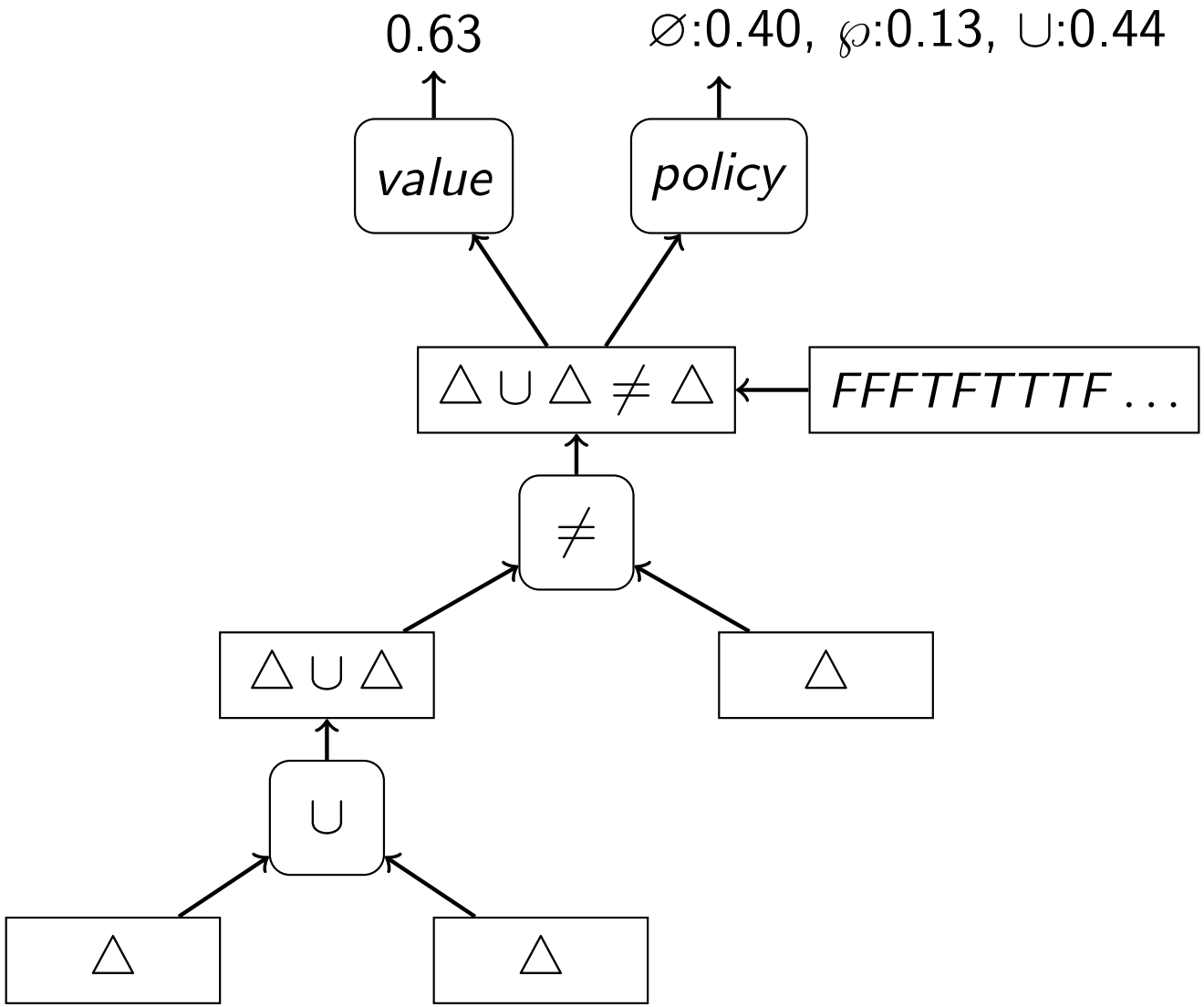
Given a graph (list of truth values), can we find a formula for it?

Reinforcement learning solution

Search tree, policy and value



Tree neural networks



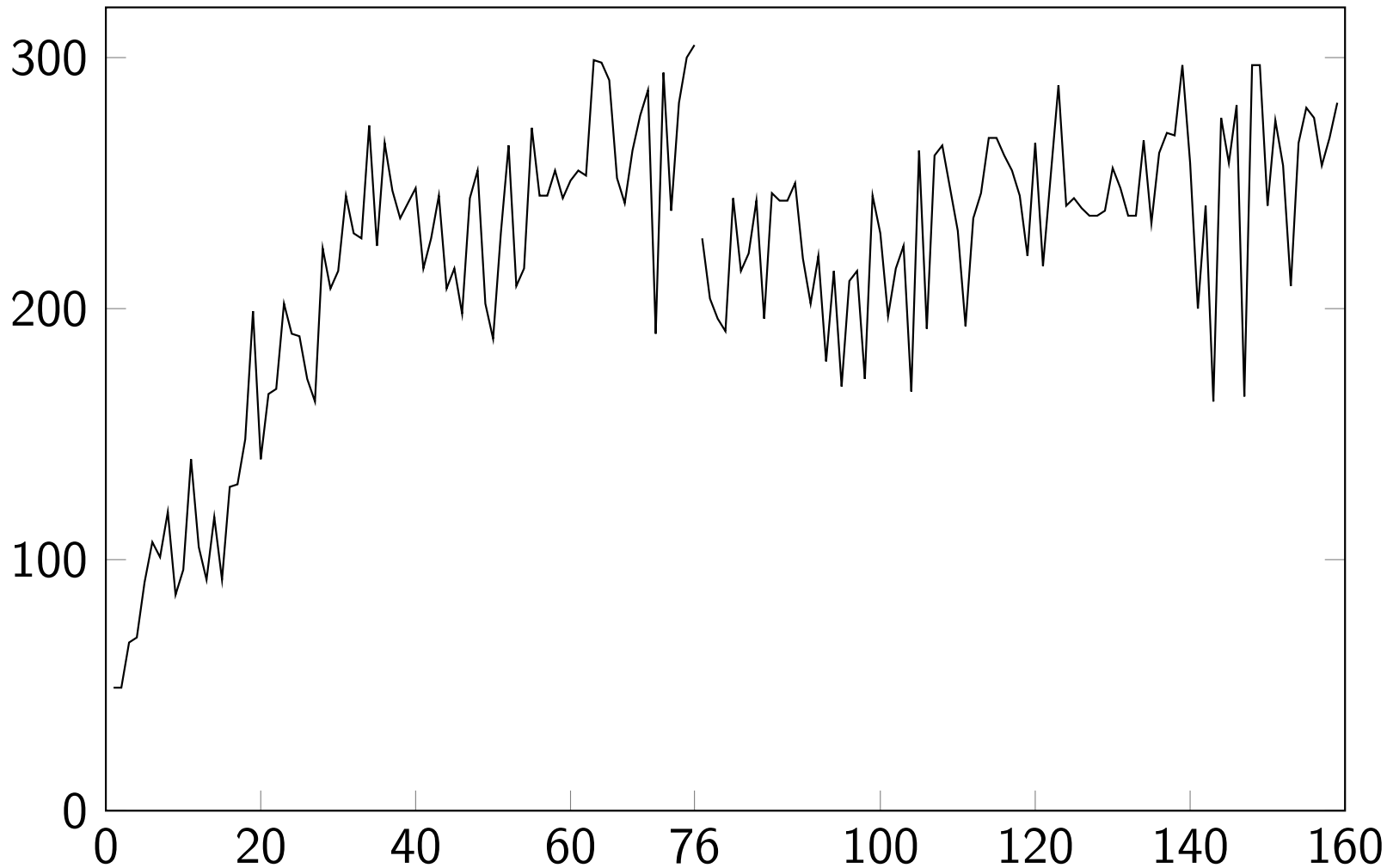
Training and testing datasets

3	4	5	6	7	8	9	10	11	12	13	14	15
6	8	22	60	88	260	472	960	638	992	1582	1056	606

Table: Number of generated graphs of each size

Level 1: 400 graphs
Level 2: 400 graphs
Level 3: 400 graphs
...

Progress of the training run



Number of successful formula synthesis (y) at generation (x)
Level 1 on the left, Level 2 on the right

Final evaluation

Abstract time limit of 50,000 search steps.

	Uniform search	Hidden-graph	Guided
Level 1, 2, 3	68, 0, 0	270, 126, 59	338, 240, 165

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$$Q(x) := \exists y \in x.\ \{y\} \neq x$$

Summary

Teach an algorithm to synthesize formulas from graphs.

Procedure for understanding a formula with one free variable:

1) Create its graph.
2) Synthesize a (new) formula.
3) Is the new formula more meaningful?

Improvements

Synthesis of combinators and Diophantine equations
(published at LPAR 2020)

1) Self-determined levels.

2) Comparison with ATPs: Vampire, E-prover.