#### A Practical Guide to Writing Tactics in Lean Daniel J. Velleman

Suppose you're writing a proof in Lean, using tactic mode, and you invoke the **assumption** tactic. Lean will search through the current context for a hypothesis that matches the goal and, if it finds one, use it to complete the proof. How does Lean know how to do this?

The answer can be found in Lean's library, in the file core/init.meta.tactic, which contains this definition:

(You can find this file at https://leanprover-community.github.io/mathlib\_docs/. On the left, under core, click on init, then meta, then tactic.)

At the moment, this definition probably doesn't mean much to you. The purpose of this guide is to explain how tactics are defined in Lean so that you will be able to read definitions like this and write definitions to create your own tactics. Here are some other resources that you may find useful:

- A Tactic Writing Tutorial, which can be found here: https://leanprover-community.github.io/extras/tactic\_writing.html
- Chapters 6 and 7 of *The Hitchhiker's Guide to Logical Verification*, by Baanen, Bentkamp, Blanchette, Hölzl, and Limperg (https://github.com/blanchette/logical\_ verification\_2021/raw/main/hitchhikers\_guide.pdf).

## **1** Preliminaries

Tactics are functions, so it may be helpful to review briefly some aspects of how functions are defined in Lean. Here is a simple function definition:

def absval (n : int) : int :=
 if n < 0 then - n else n</pre>

The definition is introduced by the keyword **def**. The name of the function is **absval**, it takes an argument **n** of type **int**, and it returns an **int**. The return value is given by the **if-then-else** expression in the second line.

A Lean function can call another function, so once we have defined the **absval** function, we can define this function:

def abs\_sum (m n : int) : int :=
 (absval m) + (absval n)

An alternative way to define this function would be:

```
def abs_sum (m n : int) : int :=
do let avm := absval m,
    let avn := absval n,
    avm + avn
```

This version gives the illusion that the function is defined by a procedural program: first absval m is computed and stored in the variable avm, then absval n is computed and stored in avn, and finally avm and avn are added. This illusion can be helpful for writing complex functions, but it is important to understand that it is only an illusion. In particular, the syntax let ... := ... cannot stand on its own; it must be followed by a comma and then another expression. Thus, for example, the following is ungrammatical:

```
def abs_sum (m n : int) : int :=
do if m < 0 then let avm := - m else let avm := m,
    if n < 0 then let avn := - n else let avn := n,
    avm + avn</pre>
```

The problem is that the **then** clause of the **if-then-else** construct is the incomplete expression **let avm** := - **m**. However, one could write:

```
def abs_sum (m n : int) : int :=
do let avm := if m < 0 then - m else m,
    let avn := if n < 0 then - n else n,
    avm + avn</pre>
```

# 2 Tactics

The art of writing tactics is called metaprogramming, and a tactic definition is introduced by the keyword **meta**. Here is a very simple tactic:

```
meta def tactic.interactive.greet : tactic unit :=
tactic.trace "Good day"
```

This defines a tactic called tactic.interactive.greet. The tactic has type tactic unit; we'll explain what this means later. The body of the tactic consists of the line tactic.trace "Good day", which tells Lean to display the message Good day to the user.

By putting this tactic in the tactic.interactive namespace, we allow a user who is writing a proof in tactic mode to invoke the tactic by typing greet. To see this tactic in action, type the definition above into a Lean file and then try this example:

```
example : true :=
begin
    greet,
    trivial
end
```

If you are using VS Code, then the word greet should have a squiggly green line under it, and if you click on it you should see the message Good day from Lean.

Let's try making the tactic a little more complicated. We begin by defining a function that has a natural number argument and returns one of three different greetings:

```
def greeting (n : nat) : string :=
match n with
    | 0 := "Good morning"
    | 1 := "Good afternoon"
    | _ := "Good evening"
end
```

Now we can rewrite the tactic like this:

```
meta def tactic.interactive.greet (n : nat) : tactic unit :=
tactic.trace (greeting n)
```

Returning to the example above, we will need to specify a natural number argument for the greet tactic. To get Lean to greet you with Good morning, we would rewrite the example like this:

```
example : true :=
begin
    greet 0,
    trivial
end
```

Change greet 0 to greet 1 to get Lean to say Good afternoon, and use greet 2 for Good evening.

Like function definitions, tactic definitions can also be written in a way that simulates procedural programming. To write a tactic definition in this simulated procedural style, we use the keyword **do** followed by a sequence of commands, separated by commas. Here is our **greet** tactic, rewritten in this style:

```
meta def tactic.interactive.greet (n : int) : tactic unit :=
do let g := greeting n,
    tactic.trace g
```

This tactic definition illustrates that a tactic can call a function and use the function's return value. A tactic can also call another tactic and use that tactic's return value, but the situation is more complicated because of some differences between tactics and ordinary functions. Unlike an ordinary function, a tactic can access the current context of the proof being written—that is, the list of goals remaining to be established to complete the proof and the hypotheses in effect for those goals. A tactic can change this context—for example, by adding or removing a hypothesis or changing a goal. And a tactic can fail—if you have been writing proofs in tactic mode, you have no doubt experienced a tactic failing. These differences make it more complicated for a tactic to call another tactic and use its return value.

Suppose, for example, that you want to define a tactic that first calls some tactic T that returns a value and then does some further processing with that returned value. The tactic T might change the proof context, and those changes must be passed on to the next step of your tactic, along with the value returned by T. And T might fail, in which case there will be no returned value and your tactic will be unable to continue.

Let's begin by illustrating the possibility that a tactic can fail. We'll rewrite our greet tactic, changing the greeting function to a tactic that may fail. This will require a number

of small changes to the definitions of greeting and greet. We'll give the new definitions first and explain the changes afterwards:

Let's run through the changes that have been made. The definition of greeting now starts with the keyword meta, and the return type is now tactic string rather than string. Think of this return type as meaning that when the tactic is applied in a proof context, it produces a box that contains a string, together with all of the other information needed to deal with the complications described earlier that arise when one tactic calls another. We'll call this a "tactic string box." For example, the box must contain information about how the proof context may have been changed by the greeting tactic, and it must contain information about whether or not the tactic failed.

Since the return type of greeting is now tactic string rather than string, the cases in the match statement must be changed. The string values corresponding to the different values of n are now preceded by return. You could think of return "Good morning" as meaning "put the string "Good morning" inside a tactic string box"; this tactic string box is then what is returned when the greeting tactic is applied in a proof context. The match statement also now has an additional case. If the argument n is anything other than 0, 1, or 2, then the tactic fails, with the failure message Illegal input.

Finally, in the greet tactic, we have written  $g \leftarrow \text{greeting n}$  rather than let g := greeting n. (In VS Code, type \l to get the symbol  $\leftarrow$ .) You can think of this command as meaning: "Invoke the tactic greeting n in the current proof context. If it succeeds, extract the string value from the resulting tactic string box, assign that value to g, and then pass on that value, along with the other information in the box, to the next step."

There's a lot going on here that has been left out of this description. If you want to know more, I recommend Chapters 6 and 7 of *The Hitchhiker's Guide to Logical Verification*. But you don't really need to know the details to be able to write tactics. You just need to know that if you want to invoke a function **f** that returns a value of type  $\alpha$ , then you should write let  $\mathbf{x} := \mathbf{f}$ , followed by further steps that may use  $\mathbf{x}$ , and if you want to invoke a tactic T that returns a value of type tactic  $\alpha$ , then you should write  $\mathbf{x} \leftarrow \mathbf{T}$ , followed by further steps that may use  $\mathbf{x}$ .

If you try out the new version of the greet tactic, you should find that if you invoke the tactic with any argument larger than 2, then the green squiggly line under greet will change to red, and Lean will display the message Illegal input. In this case the execution of the greet tactic stops as soon as the greeting tactic fails; the instruction tactic.trace g does not get executed.

As with the let  $\mathbf{x} := \mathbf{f}$  construction, it is important to keep in mind that  $\mathbf{x} \leftarrow \mathbf{T}$  cannot stand on its own, but must be followed by a comma and then further commands. So the following example would be ungrammatical:

However, if you are tempted to write this, then you can probably get the effect you intended like this:

There is an alternative notation that is sometimes useful for defining a tactic that involves a sequence of steps. If A and B are tactics, then A >> B means the same thing as do A, B. Also, if A has type tactic  $\alpha$  and B takes an argument of type  $\alpha$ , then A >>= B means the same thing as do  $x \leftarrow A$ , B x. For example, we could rewrite the greet tactic like this:

```
meta def tactic.interactive.greet (n : nat) : tactic unit :=
greeting n >>= tactic.trace
```

This saves us the trouble of giving a name to the value returned by greeting n; that nameless value is simply passed along to tactic.trace. While this notation is convenient in simple cases, in more complex tactic definitions it is probably preferable to use the do syntax.

We can now explain the type tactic unit of our greet tactic. The term unit here is a type. There is only one object of type unit, and it contains no information; it is denoted by (). The greet tactic thus produces a tactic unit box that contains this informationless object (together, of course, with the information described earlier that makes metaprogramming possible). Many tactics have no information to return, and are therefore declared to have type tactic unit. In fact, tactic.trace is a tactic with type tactic unit (it is defined in the same file core/init.meta.tactic that contains the definition of assumption). Since the last step in our greet tactic is tactic.trace, the value produced by this call to tactic.trace, which is a tactic unit box containing (), is the value returned by greet.

#### **3** Names, Expressions, and Pre-Expressions

So far we have not written a tactic that does anything useful. Before we can write useful tactics, we will need to know about three types: name, expr, and pexpr, which represent names, expressions, and pre-expressions.

Many things in Lean have names: variables, constants, functions, theorems, and so on. Examples of names include x, nat, or.intro\_left, and tactic.interactive.greet. There is a special backtick notation for defining names: if you write let n := 'x, then n will have type name, and its value will be x. You can try it out as follows:

```
meta def tactic.interactive.trace_name : tactic unit :=
do let n := 'x,
   tactic.trace (to_string n)
example : true :=
begin
   trace_name,
   trivial
end
```

The type **expr** represents an expression in Lean, such as a proposition or a proof. There is a similar backtick notation for defining expressions: the expression is enclosed in parentheses and then preceded by a backtick. Here is a simple example:

```
meta def tactic.interactive.trace_expr : tactic unit :=
do let e : expr := `(true V ¬ true),
   tactic.trace (to_string e)
example : true :=
begin
   trace_expr,
   trivial
end
```

(Note that it is necessary to explicitly give e the type expr; otherwise it will have a type that reduces to expr but is not the same as expr.) If you check the message generated by the trace\_expr tactic, you will see that it is or true (not true). This shows that the value assigned to e is the *parsed form* of the expression true  $\lor \neg$  true: it is represented as the function or applied first to the constant true and then to the result of applying the function not to the constant true.

There is an important subtlety to the backtick notation for expressions. We can illustrate it using the functions or.intro\_left and or.inl. Recall that if ha is a proof of some proposition a and b is another proposition, then or.intro\_left b ha is a proof of a  $\lor$  b; in other words, it is an expression of type a  $\lor$  b. For example, or.intro\_left false trivial has type true  $\lor$  false, because trivial is a proof of true. You can confirm this with the Lean command #check or.intro\_left false trivial, which reports the type of the expression.

It may appear that the function or.intro\_left takes two arguments, but it actually takes three. You can see this by giving the command #check @or.intro\_left, which reports that the type of or.intro\_left is  $\forall \{a : Prop\}\ (b : Prop), a \rightarrow a \lor b$ . In other words, or.intro\_left takes a proposition a, a proposition b, and a proof of a, and returns a proof of a  $\lor b$ . The curly braces around the first argument, the proposition a, indicate that this argument is *implicit*. In the expression or.intro\_left false trivial, that argument has been left out, and when Lean interprets the expression, it infers the missing first argument by a process called *elaboration*.

In many cases, Lean would be able to infer the second argument as well, so Lean has a version of the or-introduction rule in which both of the first two arguments are implicit. The command #check or.inl shows that the function or.inl has type  $\forall$  {a b : Prop}, a

 $\rightarrow$  a  $\vee$  b. Thus, if ha is a proof of a, then or.inl ha will be a proof of a  $\vee$  b, where Lean will try to infer b from the context. If you give the command #check or.inl trivial, Lean reports that the expression or.inl trivial has type true  $\vee$  ?M\_1. The ?M\_1 here is a *metavariable*; it is a placeholder, indicating that Lean was unable to infer the proposition b, so it has left it unspecified.

With that preparation, let's investigate what happens if we use these expressions in our trace\_expr tactic. First try replacing '(true  $\lor \neg$  true) in the definition of the trace\_expr tactic with '(or.intro\_left false trivial). When the trace\_expr tactic is invoked in the example proof, the message generated by the tactic.trace command is or.intro\_left true false trivial. Thus we see that Lean has elaborated the expression and inferred (correctly) that the missing first argument is true.

Now try putting in '(or.inl trivial). In VS Code, there will be a squiggly red line underneath or, and if you click on it, you will see the error message don't know how to synthesize placeholder. What's going on here is that Lean tries to elaborate the expression inside the '(...) notation when it parses the definition of the tactic. If that elaboration produces a metavariable, Lean reports an error. The lesson here is that the backtick notation for expressions can only be used for expressions whose elaboration does not produce metavariables.

Does this mean that we can't work with the expression or.inl trivial in a tactic? Not at all. We just have to tell Lean not to try to elaborate it when it parses the definition of the tactic. To do that, we put *two* backticks before the open parenthesis. Instead of an expr, we get a pexpr; this type represents a *pre-expression*, which is just an unelaborated expression. We can ask Lean to elaborate the pre-expression when the tactic is executed by invoking the tactic tactic.to\_expr, which takes an argument of type pexpr, elaborates it, and returns the resulting expr. Here's a version of our trace\_expr tactic that illustrates this:

Take a look at the message generated by this version of the trace\_expr tactic. It should be something like The pre-expression is or.inl trivial and the expression is or.inl true ?\_mlocal.\_fresh.800.574 trivial. We see that when Lean elaborated the pre-expression, it was able to infer that the first implicit argument was true, but it generated a metavariable for the second argument, which it was unable to infer. And notice something else that has happened: after the trace\_expr tactic is invoked, there are *two* goals required to finish the proof:  $\vdash$  true, which was the original goal of the example, and  $\vdash$  Prop, which is asking for a proposition to be supplied as the value of the metavariable. The trivial tactic accomplishes the first goal, and exact false takes care of the second.

Let's do one more experiment. In the last version of the trace\_expr tactic, try changing "(or.inl trivial) to "(or.inl true). Now the invocation of trace\_expr in the example will have a red squiggle under it, and if you click on it you will see the error message type mismatch at application. This indicates that the call to tactic.to\_expr failed, because true does not have the right type to be an argument of the function or.inl.

There is one more subtle point that is worth mentioning. An expression can consist of just a single constant, as in the following example:

The output is The name is not and the expression is not. It may appear that n and e are the same, but they are different: n is the name not, whereas e is an expression consisting of a single constant (whose name is not). This distinction will be important in the next section.

In our last example, the argument of tactic.trace was a string constructed by combining explicit strings, which appear in quotation marks, with expressions outside of those quotation marks that compute strings. There is a similar mechanism, called *antiquotation*, that allows us to compute expressions in a similar way. If x has type expr or pexpr, then in a double-backtick specification of a pexpr we can use the notation %%x to indicate a place where the expression that is the value of x should be inserted. Here is an example:

When the tactic is invoked with the arguments tt ff, as in the example above, o gets the value "(or.intro\_left) and d gets the value '(false). And when these values are inserted at the positions marked by %%o and %%d, the pre-expression assigned to p ends up being or.intro\_left false trivial. This pre-expression is then elaborated to produce e,

which is the expression or.intro\_left true false trivial. Try varying the arguments passed to trace\_expr to see how the results change.

Note that o had to be a pre-expression, because the elaboration of or.intro\_left or or.intro\_right would introduce metavariables, and p had to be a pre-expression, because p cannot be elaborated until execution time. In some cases, the antiquotation notation can be used in a single-backtick specification of an expr, but usually we will want to use this notation when defining a pexpr.

### 4 Accessing and Altering the Proof Context

We are finally ready to write tactics that do something useful. It may be helpful to have a simple example in front of us, so consider this example:

```
example (p q : Prop) (h1 : \neg (p \land q)) : \neg p \lor \neg q
```

When you start writing this proof in tactic mode, the state looks like this:

p q : Prop  $h1 : \neg (p \land q)$   $\vdash \neg p \lor \neg q$ 

We begin by discussing tactics in Lean's library that allow us to access the proof context. The tactic tactic.target returns an expr that is the current goal—the proposition that needs to be proven. To access a hypothesis, we must use the name of the hypothesis. The tactic tactic.get\_local takes an argument n of type name and returns an expr that is the constant whose name is n. (Actually, in this case the expr is what is called a *local constant*. As we saw in the last section, the name and the expression are not the same thing.) The tactic tactic.infer\_type takes an argument e of type expr and returns an expr that is the type of e. Here is an example that illustrates the use of these tactics:

Now we can try using the tactic in the example above:

```
example (p q : Prop) (h1 : ¬ (p ∧ q)) : ¬ p ∨ ¬ q :=
begin
    access_context
end
```

The output is The type of p is Prop, the type of h1 is not (and p q), and the goal is or (not p) (not q). (Since the types are returned as expressions, they are in parsed form.)

Of course, it would be better if we didn't have to hard code the names 'p and 'h1 in this tactic. So you might try rewriting it like this:

```
meta def tactic.interactive.access_context
    (h : name) : tactic unit :=
do H ← tactic.get_local h,
    ht ← tactic.infer_type H,
    g ← tactic.target,
    tactic.trace ("The type of " ++ (to_string h)
    ++ " is " ++ (to_string ht)
    ++ " and the goal is " ++ (to_string g))
```

Unfortunately, if we invoke the tactic with access\_context h1, we get the error message unknown identifier 'h1'. It would work to write access\_context 'h1, but we don't want users to have to know about backticks. A better idea is to tell Lean that the argument to the access\_context tactic should not be *interpreted* as an identifier; rather, we want the name that results from *parsing* the argument as an identifier to be passed to the tactic. To do this, we need to rewrite the declaration of the tactic. We can also save a little typing by opening the tactic namespace:

```
open tactic
meta def tactic.interactive.access_context
   (h : interactive.parse lean.parser.ident) : tactic unit :=
do H 	\leftarrow get_local h,
   ht 	\leftarrow infer_type H,
   g 	\leftarrow target,
   trace ("The type of " ++ (to_string h)
      ++ " is " ++ (to_string ht)
      ++ " and the goal is " ++ (to_string g))
```

Now access\_context h1 generates the output The type of h1 is not (and p q) and the goal is or (not p) (not q).

To understand what's going on here, it might be helpful note that the command **#reduce** interactive.parse lean.parser.ident produces the output name. Thus, the argument h in this tactic is a name, but by writing the type in this way we have told Lean to do the parsing we want.

Now that we can *access* the context of the proof, how do we *alter* it? The easiest way is to invoke the tactics that you are already familiar with from writing proofs in tactic mode. For instance, consider the tactic have h : t := p, which adds the hypothesis h : t, if p is a term of type t. If h is left out, then the new hypothesis is labeled this; if t is left out then it is inferred from p; and if p is left out then t becomes a new goal that must be proven. In the example above, the command have  $: \neg p \lor \neg q := not_and_distrib.mp h1$  would cause Lean to infer the new hypothesis this  $: \neg p \lor \neg q$ . (Note that you must import logic.basic for the function not\_and\_distrib to be recognized.)

The tactic have is defined in Lean's library (in the tactic.interactive namespace, of course). It has three arguments, corresponding to h, t, and p in the last paragraph, and their types are option name, option pexpr, and option pexpr, respectively. Using the have tactic, we can finally write a tactic that does something useful:

open tactic

This tactic uses backtick notation and antiquotation for pattern matching. In the first case of the match statement, if t has the form of the negation of a conjunction, then 1 and r, which have type expr, are set equal to the left and right sides of the conjunction. These expressions are then used to construct the pre-expressions that are passed to have. Similarly, the second case handles negations of disjunctions. Each call to have infers a new hypothesis labeled this by applying one of De Morgan's laws. We can use this tactic to complete the example at the beginning of this section:

```
example (p q : Prop) (h1 : ¬ (p ∧ q)) : ¬ p ∨ ¬ q :=
begin
    dm h1,
    exact this
end
```

The declaration of the have tactic (which can be found in the Lean library, in the file core/init.meta.interactive) is:

```
meta def tactic.interactive.have
  (h : interactive.parse lean.parser.ident?)
  (q<sub>1</sub> : interactive.parse
        (lean.parser.tk ":" *> interactive.types.texpr)?)
  (q<sub>2</sub> : interactive.parse
        (lean.parser.tk ":=" *> interactive.types.texpr)?)
  : tactic unit
```

We have already explained that if a tactic has an argument that is declared to have type interactive.parse lean.parser.ident, then the argument is simply a name, with the complicated type expression instructing Lean to parse the user's input to produce the name. Putting a question mark after lean.parser.ident is defined in the library file to be a shorthand for optional lean.parser.ident, so the type declaration for h means

```
(h : interactive.parse (optional lean.parser.ident))
```

As before, the command **#reduce** interactive.parse (optional lean.parser.ident) can be used to determine that the type of h is option name.

To figure out the type of q<sub>1</sub>, you can give the command **#reduce interactive.parse** (lean.parser.tk ":" \*> interactive.types.texpr). The output is expr ff, which means "expression that has not been elaborated." This is the same as pexpr; indeed, **#** reduce pexpr also produces the output expr ff. Again, adding a question mark makes

the argument optional, so the type of  $q_1$  is option pexpr. This time the instructions to the parser tell the parser to look for the token ":" followed by a pre-expression. Similarly, the type of  $q_2$  tells the parser to look for the token ":=" followed by a pre-expression, and the argument is optional, so the type of  $q_2$  is also option pexpr.

If you are trying to use another tactic from the Lean library, look at the declaration in the library file to determine what types of arguments to pass to the tactic. If necessary, you can use the **#reduce** command to help you determine the types. For example, looking in the library file **core/init.meta.interactive** we find that the declaration of the **apply** tactic is:

```
meta def tactic.interactive.apply
   (q : interactive.parse interactive.types.texpr) : tactic unit
```

This tells us that the argument has type pexpr. We will use this in our next version of the dm tactic, which will apply one of De Morgan's laws to either a hypothesis or the conclusion:

```
meta def tactic.interactive.dm
   (n : interactive.parse
      (optional (lean.parser.tk "at" *> lean.parser.ident)))
   (1 : interactive.parse
      (optional (lean.parser.tk "with" *> lean.parser.ident)))
   : tactic unit :=
match n with
  | some h := do H \leftarrow get_local h,
      t ← infer_type H,
      let label := match l with
         some h := h
        | none := `this
      end.
      match t with
        | (\neg(\% \land \% r)) := tactic.interactive.have label
                   "(¬ %%1 ∨ ¬ %%r) "(not_and_distrib.mp %%H)
        | (\neg(\%1 \lor \%r)) := tactic.interactive.have label
                   "(¬ %%1 ∧ ¬ %%r) "(not_or_distrib.mp %%H)
         [ _ := fail "De Morgan's laws don't apply"
      end
  | none := do t \leftarrow target,
      match t with
        | (\neg(\%1 \land \%r)) := tactic.interactive.apply
                      "(not_and_distrib.mpr)
        | (\neg(\%1 \lor \%r)) := \text{tactic.interactive.apply}
                      ``(not_or_distrib.mpr)
        | _ := fail "De Morgan's laws don't apply"
      end
end
```

The tactic now takes two arguments. The first argument n is an optional name, preceded by the token "at", specifying the hypothesis to which De Morgan's law should be applied. If this name is not included, the tactic operates on the goal. The second argument 1 is an optional name preceded by the token "with". If this name is supplied and the tactic is applied to a hypothesis, then the name is used instead of this as the label of the inferred statement. The tactic checks whether or not the first argument n was supplied. If so, it uses the **have** tactic as before; if not, it uses the **apply** tactic to rewrite the goal using one of De Morgan's laws.