A ForTheL-like CNL for Lean

Peter Koepke

June 7, 2019

This is a ForTheL version of some example text for a CNL for Lean proposed by Tom Hales at Big Proof 2019.

Preliminaries on Types

[synonym type/-s]

Signature 1. *A* type *is a class. Let C stand for a class. Let A stand for a type. Let $a : t$ stand for a is an element of t. Let $a \in t$ stand for a is an element of t.*

Signature 2. *C is finite is an atom.*

Preliminaries on Numbers

[synonym number/-s]

Signature 3. *A* number *is a notion. Let m, n, p, q denote numbers.*

Signature 4. *$m * n$ is a number.*

Signature 5. *$m - 1$ is a number.*

Definition 1. *A* divisor *of n is a number m such that $n = m * a$ for some number a. Let m* divides *n stand for m is a divisor of n.*

Signature 6. *p^m is a number.*

Signature 7. *Assume C is finite. $|C|$ is a number. Let the* order *of C stand for $|C|$. Let the* size *of C stand for $|C|$.*

Signature 8. *p is* prime *is an atom.*

Signature 9. *The* multiplicity *of p in n is a number.*

Preliminaries on Groups

[synonym group/-s]

Signature 10. *A* group *is a type. Let* G, H *denote groups.*

Signature 11. *Let* $x, y : G$. $x *^G y$ *is an element of* G.

Signature 12. *Let* $x : G$. $x^{-1,G}$ *is an element of* G.

Definition 2. *A* subgroup *of* G *is a group* H *such that every element of* H *is an element of* G.

Axiom 1. *Let* G *be a finite group. Every subgroup of* G *is finite.*

Sylow Subgroups

Definition 3 (Conjugate)**.** *Assume* $g : G$. *Assume that* H *is a subgroup of* G. *The* conjugate *of* H *by* g *in* G *is the subgroup* K *of* G *such that for all elements* x *of* G $x \in K \Leftrightarrow (g *^G x) *^G g^{-1,G} \in K$.

Definition 4 (Normalizer)**.** *Assume that* H *is a subgroup of* G. *The* normalizer *of* H *in* G *is the subgroup* N *of* G *such that for all elements* x *of* G $x \in N$ *iff for all elements* h *of* H $(x^{-1,G} *^G h) *^G x \in H$.

Let G, P, Q denote finite groups. Let p denote a prime number.
[synonym subgroup/-s]

Definition 5 (Sylow)**.** *A* Sylow subgroup *of* G *for* p *is a subgroup* P *of* G *such that* $|P| = p^m$ *where* m *is the multiplicity of* p *in* $|G|$.

Definition 6. $\mathrm{S}yl(p, G) = \{R \mid R \textit{ is a Sylow subgroup of Gforp}\}$.

Axiom 2. $\mathrm{S}yl(p, G)$ *is finite.*

Definition 7. $n(p, G)$ *is the size of* $\mathrm{Syl}(p, G)$.

Axiom 3 (Sylow1)**.** *There exists a Sylow subgroup of* G *for* p.

Axiom 4 (Sylow2)**.** *If* P, Q *are Sylow subgroups of* G *for* p *then there exists* $g : G$ *such that* Q *is the conjugate of* P *by* g *in* G.

Axiom 5 (Sylow3a)**.** *Assume that* $|G| = q * (p^m)$. *Then* $n(p, G)$ *divides* q.

Axiom 6 (Sylow3b)**.** p *divides* $n(p, G) - 1$.

Axiom 7 (Sylow3c)**.** *Let* $\mathrm{N}orm$ *be the normalizer of* P *in* G *where* P *is a Sylow subgroup of* G *for* p. *Then* $n(p, G) * |\mathrm{N}orm| = |G|$.